March 2022

YBMA News

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No. 3

The Newsletter of the Yorkshire Branch of the Mathematical Association

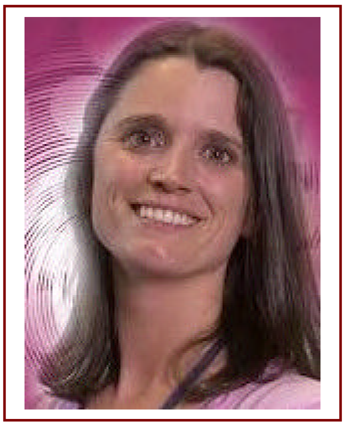
W P Milne Lecture for Sixth Formers

Wednesday 30th March 2022 at 2:30pm at the University of Leeds

To Infinity and Beyond*

Dr Katie Chicot

(Open University/MathsWorldUK)



Dr Katie Chicot is a Senior Lecturer in Mathematics and Statistics at the Open University and Chief Executive Officer of MathsWorldUK.

MathsWorldUK opened the UK's first ever Maths Discovery Centre **MathsCity** in Leeds last October (for details, go to https://mathscity.co.uk).

Katie's interest in all things mathematical stems from a love of investigation and challenge. Tackling mathematical problems and encouraging others to engage with mathematical investigations are the cornerstones of her work. She is a graduate of the University of Leeds where she gained a BA in Mathematics and Philosophy and a PhD in Mathematical Logic.

Katie has used all sorts of methods for communicating mathematics including co-creating the series 'Patterns of life' for OU iTunes , captaining the OU's team on BBC2's Beat the Brain, participating in Facebook events and more seriously as academic consultant to BBC Radio 4's 'More or Less' .

"The infinitely large and the infinitely small are mind-blowing concepts that have helped mathematicians to solve some very real, and finite, problems.

"In this lecture I will explore the mysteries and misconceptions of infinity, from ancient puzzles to some of the latest mathematical research, taking you to infinity … and beyond."

The lecture is open to anyone interested; not just sixth formers. Please bring it to the attention of anyone who might be interested in attending.

Anyone who wishes to attend this lecture must register in advance by sending a message to: a.slomson@leeds.ac.uk

This is because we need to control the number of people attending, and so that we can let you know if there needs to be a last minute change in the arrangements.

*Please note that this is a change from the originally announced lecture by Dr James Cranch. He has explained that "The University and College Union (UCU) has called a sequence of rolling strikes across March and April, continuing their dispute over the planned downgrading of the university pension scheme, the repeated below-inflation pay rises, and working conditions. The University of Sheffield will be on strike 28th March-1st April, and I have been instructed to withdraw my labour on that week, including my outreach activities. I'm very sorry that this means I am unable to deliver the 2022 Milne lecture as planned on the 30th March." The University of Leeds strike dates do not affect this event.

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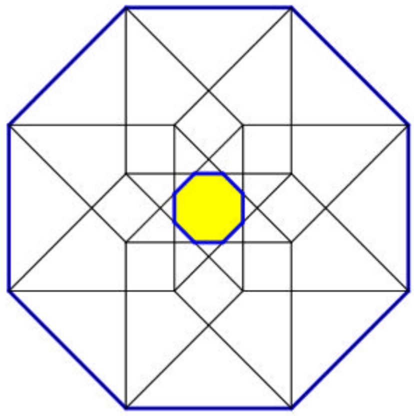
See the next page for *Mathematics in the Classroom*

The area of an octagon



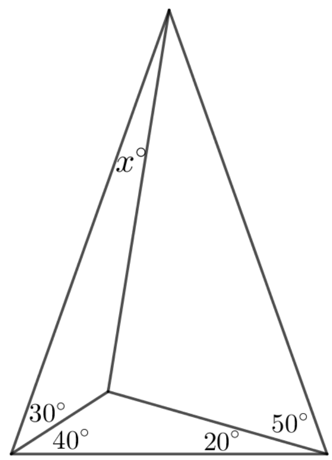
The large regular octagon shown with blue edges has sides of length 1.

A square has been constructed, inside the octagon, on each edge of the octagon.

The small yellow octagon is the region which is inside the large octagon, but not inside any of the squares.

What is the area of the yellow octagon?

Bernard Murphy's Problem

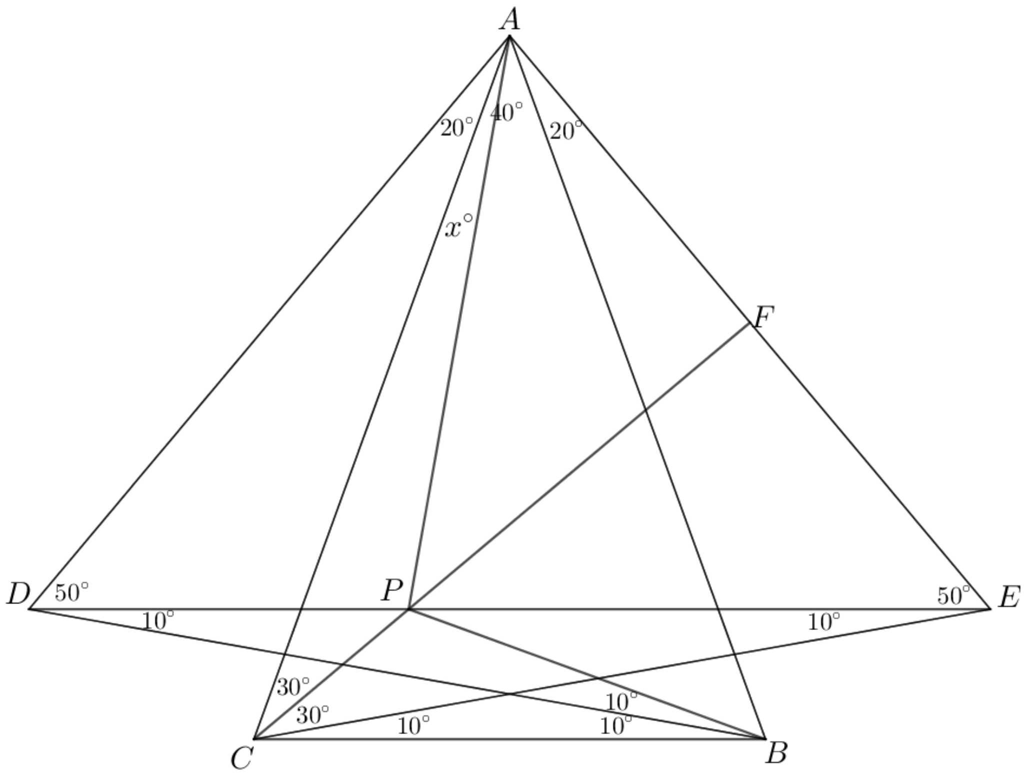


In the January 2022 Newsletter we gave the problem posed by Bernard Murphy of finding the angle $x°$ in the diagram shown on the right.

Bernard has supplied one solution to his problem. Bill Bardelang provided a solution to a generalized version of the problem. Both solutions are given below.

It will be seen that both proofs are very elegant. Each proof uses an ingenious construction, but once the construction has been found, the rest of the proof uses only basic facts about triangles.

Bernard Murphy's Solution



We let the vertices of the triangle be labelled A, B and C as shown in the diagram. Let D and E be points such that the triangles ABD and ACE are equilateral. We let F be the midpoint of AE. Let P be the point where CF meets DE.

Because the triangle ACE is equilateral and F is the midpoint of AE, the triangles FCE and FCA are congruent. Therefore $\angle FCE = \angle FCA = 30^\circ$.

Therefore in the triangles CAP and CEP, we have $\angle ACP = \angle ECP = 30^\circ$, $AC = EC$ as they are two sides of an equilateral triangle, and the side PC is common. It follows that the triangles are congruent. Therefore

$$\angle CAP = \angle CEP. \qquad (1)$$

Since $\angle CAB = 40^\circ$ and $\angle DAB = 60^\circ$, it follows that $\angle DAC = 20^\circ$. Therefore $\angle DAE = \angle DAC + \angle CAE = 20^\circ + 60^\circ = 80^\circ$. Also $AE = AC = AB = AD$. Therefore DEF is an isosceles triangle. Hence $\angle ADE = \angle AED = 50^\circ$. Therefore

$$\angle CEP = \angle CEA - \angle DEA = 60^\circ - 50^\circ = 10^\circ \qquad (2)$$

Also, similarly, $\angle BDE = 10^\circ$.

By (1) and (2), $\angle CAP = 10^\circ$.

We also have

$$\angle PCB = \angle ACB - \angle ACP = 70^\circ - 30^\circ = 40^\circ. \qquad (3)$$

It follows that $\angle ECB = \angle PCB - \angle PCE = 40^\circ - 30^\circ = 10^\circ = \angle CEP$. Hence DE is parallel to CB. Therefore

$$\angle CBD = \angle BDE = 10^\circ. \qquad (4)$$

We also have $\angle DAP = \angle DAC + \angle CAP = 20^\circ + 10^\circ = 30^\circ$. Therefore $\angle PAB = 30^\circ$. Since $AD = AB$ and the side AP is common to the triangles DAP and BAP, it follows that these triangles are congruent. Therefore $PD = PB$. Hence

$$\angle DBP = \angle BPD = 10^\circ. \qquad (5)$$

By (4) and (5)

$$\angle CBP = \angle CBD + \angle DBP = 10^\circ + 10^\circ = 20^\circ. \qquad (6)$$

It follows that point P coincides with the point inside the triangle given in the question.

Therefore $x = 10$.

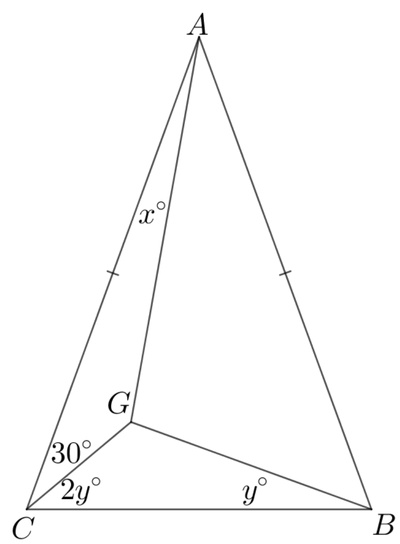
Bill Bardelang's Solution

Here, we suppose that the triangle ABC is isosceles with $AB = AC$, and that $\angle ACG = 30^\circ$ and $\angle GCB = 2\angle GBC$.

Let $\angle CAG = x^\circ$ and $\angle GBC = y^\circ$.

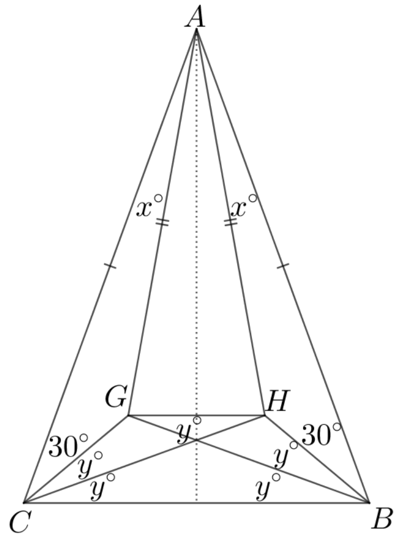
The problem is to find x in terms of y.

Thus Bernard Murphy's problem is the case where $y = 20$.



Solution

Let H be the reflection of the point G in the axis of symmetry of the triangle ABC. By the symmetry of the figure, $\angle HAB = \angle GAC = x^\circ$ and $\angle ABH = \angle ACG = 30^\circ$.



Also,

$$\angle BCH = \angle CBG = y^\circ,$$

and therefore,

$$\angle GCH = y^\circ$$

Because GH is parallel to CB,

$$\angle CHG = \angle BCH = y^\circ.$$

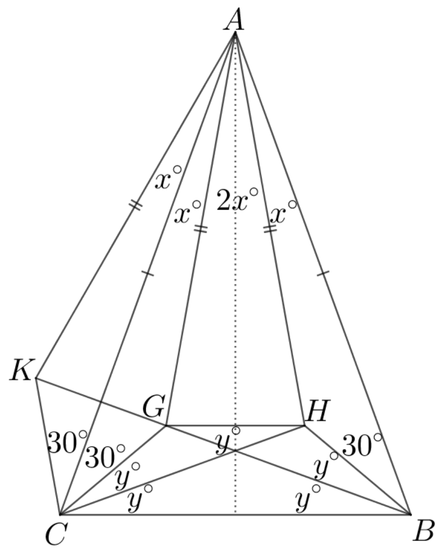
Hence $\angle GCH = \angle CHG$ and therefore $GH = GC$.

We now let K be the reflection of G in the line AC.

It follows that, $GC = KC$ and $\angle KCA = \angle ACG = 30^\circ$.

Therefore $\angle KCG = 60^\circ$. Hence, as $GC = KC$, the triangle KCG is equilateral and hence $KG = GC = GH$. Since $AH = AG = AK$ it follows that the triangles AGH and AKG are congruent.

Also $\angle KAC = \angle GAC = x^\circ$ and therefore $\angle GAH = \angle KAG = 2x^\circ$.



Now from the angles in the triangle ABC we have

$4x + (30 + 2y) + (30 + 2y) = 180$,

from which it follows that $x + y = 30$.

In the case $y = 20$, this gives $\angle GAC = 10^\circ$.

Solution to our New Year Crossnumber

On the right is the solution to the Crossnumber in the January Newsletter.

[1] 1	[2] 4	[3] 2	8	[4] 7	[5] 6	[6] 3
[7] 9	0	2		[8] 3	5	9
[9] 4	5				[10] 5	9
[11] 8	8	8		[12] 2	3	1
[13] 7	5				[14] 7	6
1						8
[15] 7	[16] 2	9		[17] 7	[18] 2	0
[19] 1	5				[20] 1	0

YBMA Officers 2021-2022

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